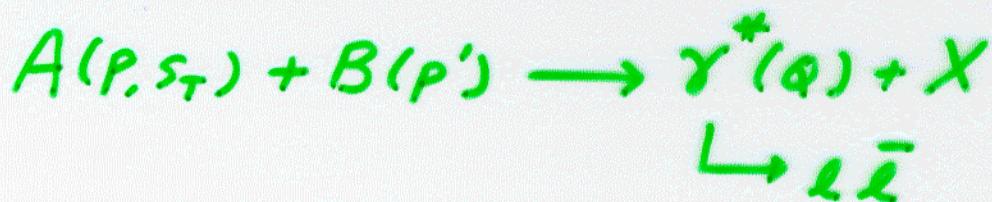


Single Transverse Spin Asymmetries in Drell-Yan lepton's angular distribution

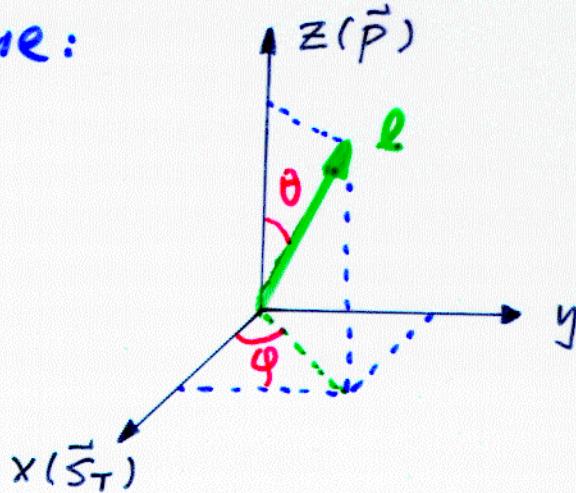
Based on Work by

D. Boer and J. Qiu

* Process:



* Frame:



$\vec{Q} = 0$ frame

$$\begin{aligned} \vec{Q}^\mu &= \frac{Q}{2} [1, \sin\theta \cos\phi, \\ &\quad \sin\theta \sin\phi, \cos\theta] \end{aligned}$$

* Observable:

$$\frac{d\sigma(s_T)}{dQ^2 d\Omega}$$

* Asymmetry:

$$\frac{1}{2} \left[\frac{d\sigma(s_T)}{dQ^2 d\Omega} - \frac{d\sigma(-s_T)}{dQ^2 d\Omega} \right]$$

$$A_N = \frac{\frac{1}{2} \left[\frac{d\sigma(s_T)}{dQ^2 d\Omega} + \frac{d\sigma(-s_T)}{dQ^2 d\Omega} \right]}{\frac{1}{2} \left[\frac{d\sigma(s_T)}{dQ^2 d\Omega} - \frac{d\sigma(-s_T)}{dQ^2 d\Omega} \right]}$$

* Existing Calculations :

- Light-Cone gauge [Hammon, Teryaer, Schäfer, PLB(97)]

$$A_N^{(HTS)} = \left[\frac{\sin 2\theta \sin \Phi}{1 + \cos^2 \theta} \right] \frac{g}{Q} \frac{[T(x, x) - \pi \frac{d}{dx} T(x, x)]}{q(x)}$$

- Light-Cone gauge [Boer, Mulder, Teryaer, PRD57(98)]

$$A_N^{(BMT)} = \left[\frac{\sin 2\theta \sin \Phi}{1 + \cos^2 \theta} \right] \cdot \frac{g}{Q} \cdot \frac{T(x, x)}{q(x)}$$

- Key difference :

Derivative term of $T(x, x)$.

- Twist-3 quark-gluon correlation (Qiu & Sterman '91)

$$T(x, x) \equiv -T_F^{\nu}(x, x)$$

$$T_F^{\nu}(x, x) = \int \frac{dy^-}{2\pi} e^{ixp^+y_i^-} \langle p, s_T | \bar{\psi}(0) \frac{r^+}{z}$$

$$* \left[\int dy_i^- \epsilon^{\sigma \tau \mu \nu} F_{\sigma}^+(y_i^-) \right] \psi(y_i^-) |p, s_T\rangle$$

— also responsible for single transverse spin asym. in inclusive π production

* Our analytical result:

— in both Light-cone and Covariant gauge

$$A_N = 2 \left[\frac{\sin 2\theta \sin \varphi}{1 + \cos^2 \theta} \right] \frac{g}{Q} \cdot \frac{\sum_q e_q^2 \Phi_{\bar{q}}(x') \otimes T_q(x, x)}{\sum_q e_q^2 \Phi_{\bar{q}}(x') \otimes \Phi_q(x)}$$

$$\rightarrow 2 \left[\frac{\sin 2\theta \sin \varphi}{1 + \cos^2 \theta} \right] \frac{g}{Q} \cdot \frac{T(x, x)}{q(x)}$$

- No $\times \frac{d}{dx} T(x, x)$ term.
- A factor of 2 Larger.

* Why?

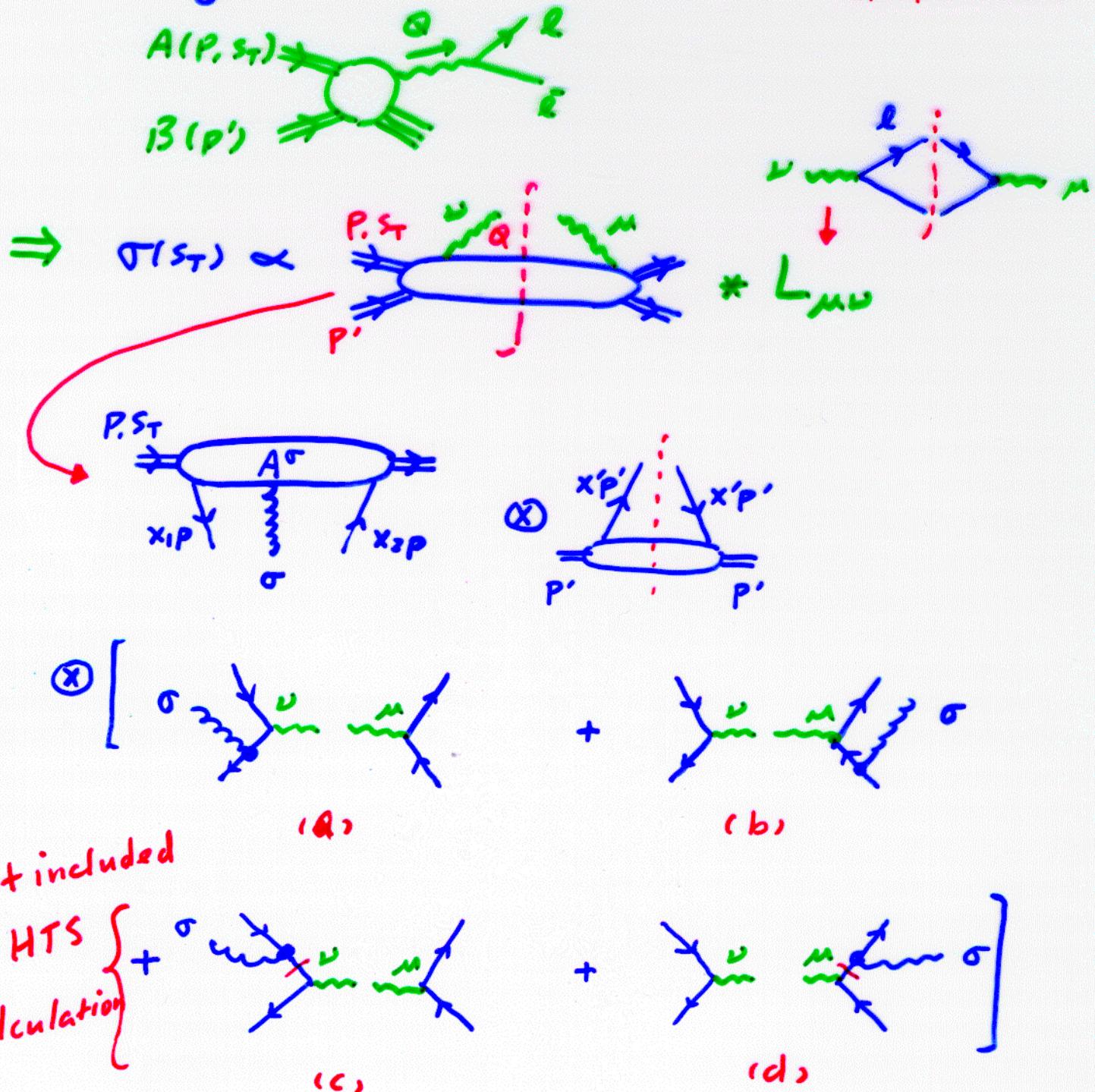
- Explicit EM current conservation
 \Rightarrow the factor of 2
- Color gauge invariance at lowest order
 \Rightarrow kill the $\times \frac{d}{dx} T(x, x)$ term
- $\times \frac{d}{dx} T(x, x)$ term contributes at high order

Calculation with Light-cone gauge in QCD

— Compare to that of HTS & BMT

* Factorization:

Qiu & Sterman '91



* Single Spin Matrix element :

- $T^\sigma(x_1, x_2) = \frac{P_\nu S_\sigma}{x_1 p - x_2 p}$

$$\propto \langle 1 \bar{\psi} A^\sigma \psi | \rangle$$

Not a color gauge invariant operator.

$$\Rightarrow \langle 1 \bar{\psi} F^{+\sigma} \psi | \rangle \propto T_F^{IV}(x_1, x_2)$$

- In Light-cone gauge, $\sigma \sim \perp$ leading Contrib.

$$\Rightarrow A^\sigma(y_2^-) \rightarrow \frac{i}{(x_2 - x_1) P^+} \left(\frac{\partial}{\partial y_2^-} A^\sigma(y_2^-) \right)$$

$$\rightarrow \frac{i}{(x_2 - x_1) P^+} F^{+\sigma}(y_2^-)$$

* Partonic part : Cancel

Pole \rightarrow necessary "i" to get a real A_N

$$H_{(a+b)}^{\mu\nu\sigma} = \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right] + \left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right]$$

$$= e_q^2 \cdot g \cdot \left[\frac{x_2 - x_1}{x_2 - x_1 + i\epsilon} \right] \left\{ g^{\mu\sigma} \frac{p^\nu}{x'} \delta(x_2 - \frac{Q^2}{x's}) + g^{\nu\sigma} \frac{p^\mu}{x'} \delta(x_1 - \frac{Q^2}{x's}) \right\}$$

But, $q_\nu H^{\mu\nu\sigma} \neq 0$

$$q_\nu H^{\mu\nu\sigma} \neq 0$$

* The missing term:

$$H_{(c+d)}^{uv\sigma} = \left[\text{Diagram } a + \text{Diagram } b \right] \quad \begin{matrix} \text{Special propagator} \\ \text{Qiu '90} \end{matrix}$$

$$= e_q^2 \cdot g \cdot \left[g^{u\sigma} \left(-\frac{P'^u}{x_2} \right) \delta(x_2 - \frac{Q^2}{x's}) + g^{v\sigma} \left(-\frac{P'^v}{x_1} \right) \delta(x_1 - \frac{Q^2}{x's}) \right] \left[\frac{x_2 - x_1}{x_2 - x_1 + i\epsilon} \right]$$

↑ Mueller & Qiu '81

* Combined particle part:

$$H^{uv\sigma} = e_q^2 \cdot g \cdot \left[\frac{x_2 - x_1}{x_2 - x_1 + i\epsilon} \right] \left\{ g^{u\sigma} \left(\frac{P^u}{x'} - \frac{P'^u}{x_2} \right) \delta(x_2 - \frac{Q^2}{x's}) + g^{v\sigma} \left(\frac{P^v}{x'} - \frac{P'^v}{x_1} \right) \delta(x_1 - \frac{Q^2}{x's}) \right\}$$

* Phase needed for the A_N :

$$\overline{\frac{1}{x_2 - x_1 + i\epsilon}} \longrightarrow -2\pi i \delta(x_2 - x_1)$$

* Hadronic Contribution to A_N :

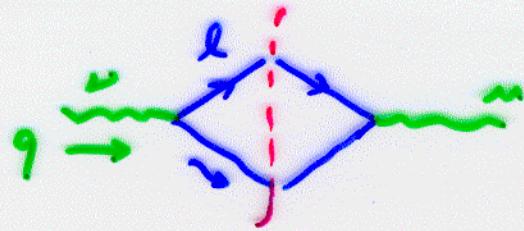
$$\int dx_1 dx_2 T_\sigma(x_1, x_2) H^{uv\sigma}(x_1, x_2)$$

$$\Rightarrow \int dx \delta(x - \frac{Q^2}{x's}) T_F^{uv}(x, x) \left(\frac{g}{x's} \right) \in S_{T\sigma nn}$$

$$\times \left[g^{u\sigma} \left(\frac{P^u}{x'} - \frac{P'^u}{x} \right) + g^{v\sigma} \left(\frac{P^v}{x'} - \frac{P'^v}{x} \right) \right]$$

Explicit EM current conservation, $q^u = x P^u + x' P'^u$

* Leptonic Part :



$$L_{\mu\nu} = 4 \left[\ell_M (\bar{q} - \ell)_\nu + (\bar{q} - \ell)_m p_\nu - \ell \cdot \bar{q} g_{\mu\nu} \right]$$

* Asymmetry :

$$\left[g^{\mu\nu} \sigma \left(\frac{P''}{x'} \right) + g^{\nu\mu} \sigma \left(\frac{P''}{x'} \right) \right] L_{\mu\nu}$$

$$= \left[g^{\mu\nu} \sigma \left(-\frac{P''}{x} \right) + g^{\nu\mu} \sigma \left(-\frac{P''}{x} \right) \right] L_{\mu\nu}$$

- ⇒ • Our result is a factor of 2 Larger
• No derivative term!

$\frac{i}{x_2 - x_1} \cdot \frac{\gamma \cdot (x_2 - x_1) P - \gamma \cdot (x' p')}{(x_2 - x_1) x' s + i\epsilon}$

$\gamma \cdot p' \rightarrow$ Picks up $\gamma \cdot (x_2 - x_1) P$ term

$$\Rightarrow \frac{i}{x_2 - x_1} \cdot \frac{x_2 - x_1}{x_2 - x_1 + i\epsilon} \rightarrow \frac{1}{x_2 - x_1 + i\epsilon} \Rightarrow T(x, x)$$

- Direct γ or high order D-Y :

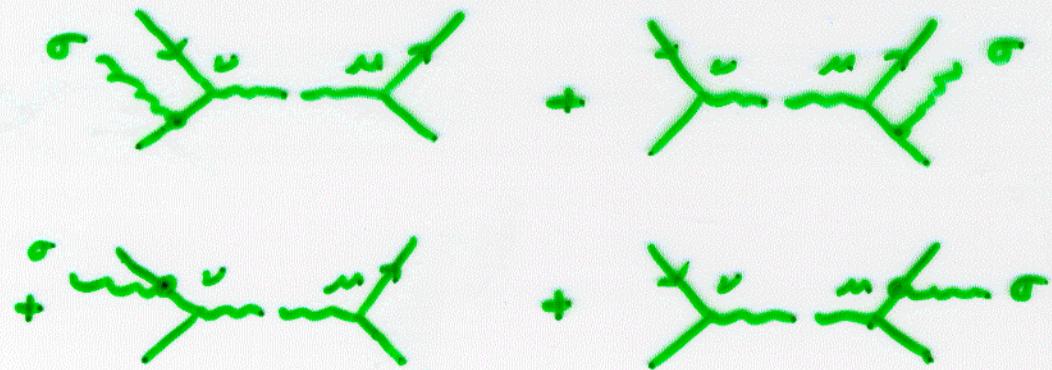
$$\Rightarrow \left(\frac{1}{x_2 - x_1 + i\epsilon} \right)^2 \Rightarrow x \frac{d}{dx} T(x, x)$$

k
has transverse component!

Calculation in Covariant gauge in QCD

— Cross Check (Method of Qiu & Sterman '91)

* Same basic Feynman diagrams:



* Matrix element in Covariant gauge:

$$T^\sigma(x_1, x_2) \propto P^\sigma \quad \text{Leading Contribution}$$

$$A^\sigma \rightarrow A^\sigma \cdot n_\sigma = A^+$$

$$\Rightarrow \text{Need } k_T^\sigma A^+(y) \rightarrow F^{P+}(y)$$

- Let the gluon carry a small k_T ,

$$\rightarrow H^{\mu\nu}(x_1, x_2, k_T)$$

- Expand $H^{\mu\nu}(x_1, x_2, k_T)$ at $k_T = 0$

$$H^{\mu\nu}(x_1, x_2, k_T) \approx \underbrace{H^{\mu\nu}(x_1, x_2, k_T=0)}_{\substack{\text{twist-2} \\ \text{eikonal line}}} + \underbrace{\frac{\partial H^{\mu\nu}}{\partial k_T^\mu} \cdot (k_T^\mu)}_{\substack{\text{partonic part} \\ \text{for twist-3}}} + \dots$$

* Partonic part from (a+b):

- No $\frac{1}{x_2 - x_1}$ from matrix element

- $P_\sigma \left(\sigma \begin{array}{c} \nearrow \gamma \\ \swarrow \gamma \\ \text{---} \end{array} \nu \right) \sim \gamma \cdot P \left[\frac{\gamma \cdot (x_2 - x_1) P - \gamma \cdot x' p'}{(x_2 - x_1) x' s + i\epsilon} \right]$

Picks up the pole.

$$\frac{\partial H^{\mu\nu}}{\partial k_T^\rho} (a+b) = e_q^2 \cdot g \cdot \left[\frac{1}{x_2 - x_1 + i\epsilon} \right] \left\{ g^{\mu\rho} \frac{P^\nu}{x'} \delta(x_2 - \frac{Q^2}{x's}) + g^{\nu\rho} \frac{P^\mu}{x'} \delta(x_1 - \frac{Q^2}{x's}) \right\}$$

- Same as that of BMT
- Same as the result from Light-cone gauge
 \Rightarrow a factor of 2 smaller

* Contribution from (c+d):

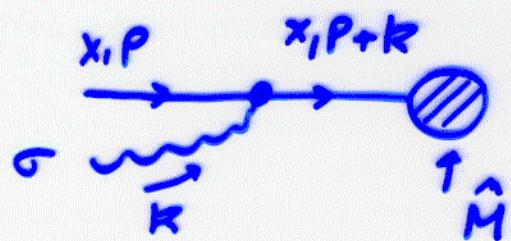
$$P_\sigma \left(\sigma \begin{array}{c} \nearrow \gamma \cdot P \\ \swarrow \gamma \\ \text{---} \end{array} \nu \right) \propto \gamma \cdot P \gamma^\sigma \cdot P_\sigma = P^2 = 0$$

Not contribute ?!

But, the propagator vanishes as $\frac{1}{p^2}$.

$\Rightarrow \frac{P^2}{p^2}$ or $\frac{0}{0}$ situation!

* Extract the twist-3 Contribution from the $\frac{\partial}{\partial}$:
 — piece proportional to k_T



$$= \text{Tr} \left[\hat{M} \frac{\gamma \cdot (x_1 P + k)}{(x_1 P + k)^2 + i\epsilon} \gamma^\sigma \gamma \cdot P \right]$$

$$\bullet \Rightarrow \text{Tr} \left[\hat{M} \frac{\gamma \cdot k_T}{(x_1 P + k)^2 + i\epsilon} \gamma^\sigma \gamma \cdot P \right] \cdot P_\sigma \rightarrow P^2$$

$$\bullet (x_1 P + k)^2 = x_1 z P \cdot k + k^2$$

$\underbrace{k^2}_{k_T^2}$ Not Contribute
to twist-3

$$\approx x_1 z P \cdot k$$

Let $k = (x_2 - x_1)P + k_T$,

denominator $= (x_2 - x_1) x_1 \cdot z P^2$

Numerator $= \text{Tr} [\hat{M} \gamma \cdot k_T] P^2 \xrightarrow{\text{cancel}}$

$$\Rightarrow \left(\begin{array}{c} x_1 P \\ \sigma \\ \hbar \end{array} \right)_{\text{twist-3}} P_\sigma = \text{Tr} [\hat{M} \gamma \cdot k_T] \frac{1}{(x_2 - x_1) + i\epsilon} \left(\frac{1}{z x_1} \right)$$

* Contribution from (c+d):

$$\frac{\partial H^{\mu\nu}}{\partial k_T P}(c+d) = e_q^2 \cdot g \cdot \left[\frac{1}{x_2 - x_1 + ie} \right] \left\{ g^{\mu\rho} \left(-\frac{P'}{x_2} \right) \delta(x_2 - \frac{Q^2}{x_2}) \right.$$

$$\left. + g^{\nu\rho} \left(-\frac{P'^\nu}{x_1} \right) \delta(x_1 - \frac{Q^2}{x_1}) \right\}$$

Same as the result in
Light-cone gauge.

* Combined Contribution in Covariant gauge :

$$\frac{\partial H^{\mu\nu}}{\partial k_T P} = e_q^2 \cdot g \cdot \left[\frac{1}{x_2 - x_1 + ie} \right] \left\{ g^{\mu\rho} \left(\frac{P^\nu}{x'} - \frac{P'^\nu}{x_1} \right) \delta(x_2 - \frac{Q^2}{x'_S}) \right.$$

$$\left. + g^{\nu\rho} \left(\frac{P^M}{x'} - \frac{P'^M}{x_1} \right) \delta(x_1 - \frac{Q^2}{x'_S}) \right\}$$

- Explicit EM current conservation
- Same result as that of Light-cone gauge.
- The missing factor of 2 for BMT
is due to missing contribution of
diagrams (c+d).

* Size of the Asymmetry

$$A_N = \left[\frac{\sin 2\theta \sin \varphi}{1 + \cos^2 \theta} \right] 2 \cdot \frac{\sqrt{4\pi \alpha_s}}{Q}.$$

$$\# \frac{\sum_q e_q^2 \Phi_{q\bar{q}}(x') \otimes T_q(x, x)}{\sum_q e_q^2 \Phi_{q\bar{q}}(x') \otimes \Phi_q(x)}$$

- $2 \sqrt{4\pi \alpha_s} / Q \simeq 2 \cdot \frac{2}{Q} = \begin{cases} 1 & \text{for } Q = 4 \text{ GeV} \\ 2 & Q = 2 \text{ GeV} \\ \text{above or below } \Upsilon/\psi \end{cases}$

- $\frac{\sin 2\theta \sin \varphi}{1 + \cos^2 \theta} \leq 0.71 \quad (\text{see figure})$

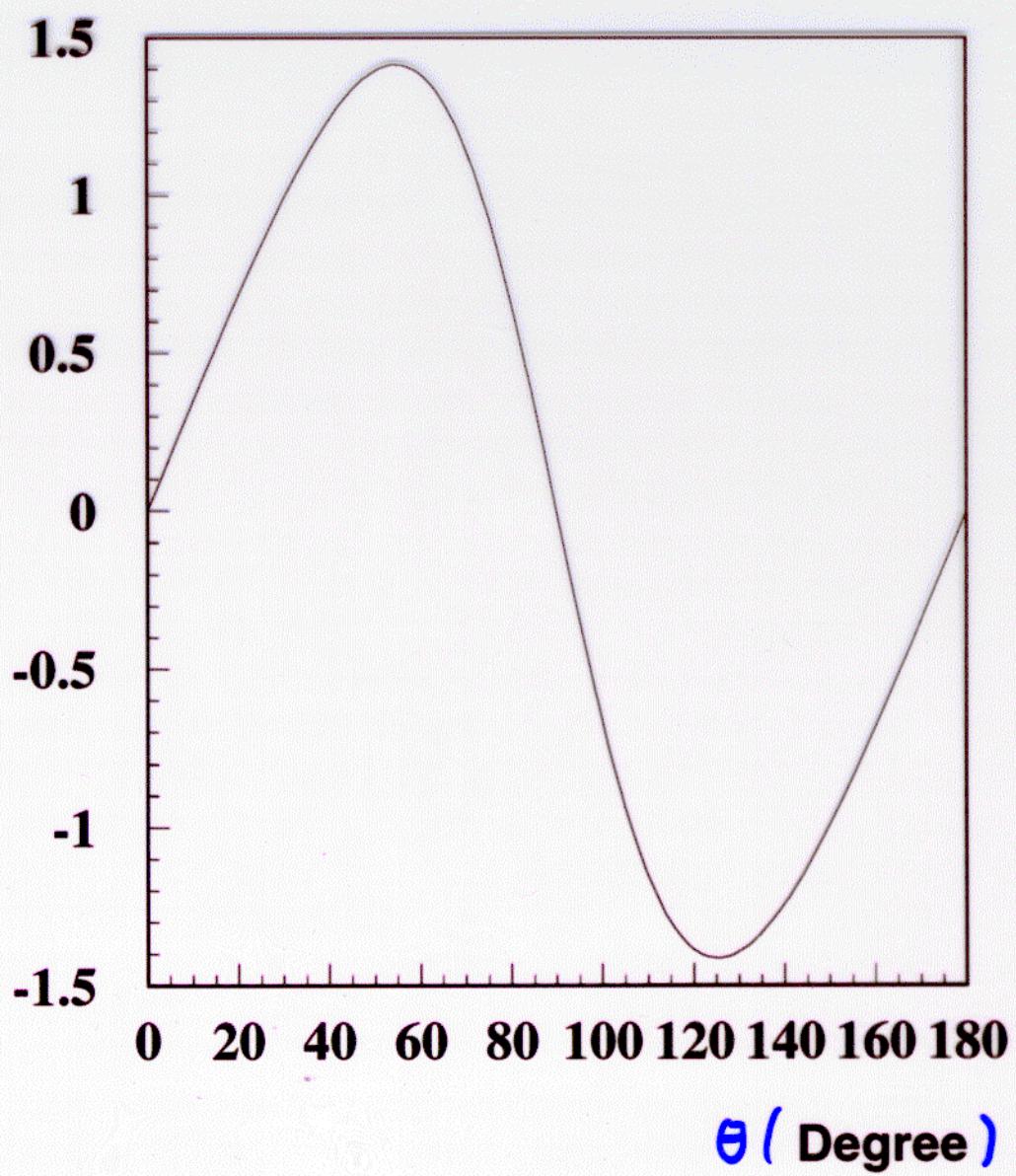
- $\frac{\sum_q e_q^2 \Phi_{q\bar{q}}(x') \otimes T_q(x, x)}{\sum_q e_q^2 \Phi_{q\bar{q}}(x') \otimes \Phi_q(x)} < \frac{\lambda}{2} \sim \frac{1}{2} \Lambda_{\text{QCD}}$

$T_q(x, x) \sim \lambda \Phi_q(x)$
 λ
 $q \leftrightarrow \bar{q}$

$$\Rightarrow A_N < 0.7 * 2 * \frac{\Lambda_{\text{QCD}}}{Q} \sim \frac{\Lambda_{\text{QCD}}}{Q}$$

Small !

$$2 \left[\frac{\sin 2\theta}{1 + \cos^2 \theta} \right]$$



Conclusions

- We derive the single transverse Spin Asymmetry for Drell-Yan lepton angular distribution.
- We explain the difference between our result and two published calculations.
- The key to the difference is the EM current conservation.
- As expected, asymmetry A_N for Drell-Yan inclusive cross section is small, $A_N \propto \frac{\Lambda_QCD}{Q}$
 - Consistent with the general conclusion Qin & Sterman '91
 - $A_N \sim \frac{\Lambda_QCD}{Q}$ is small for inclusive cross section (twist-3 effect).
 - A_N can be large for differential cross section (kinematic suppression to twist-2, enhancement to twist-3)
 - e.g. pion asymmetry at large x_F .